Application of Soft Computing

Unit 3

Crisp Set or Conventional Set or Classical Sets

- Definition of set
- Cardinality of Set
- Subset
- Superset
- Power of a set
- Operations on Crisp Sets
- Properties of Crisp Set

Operations on Crisp Set

- Complement
- Union
- Intersection

Properties of CRISP SETS

Involution	$\overline{\overline{A}} = A$
Commutativity	$A\cup B=B\cup A,\ A\cap B=B\cap A$
Associativity	$A\cup B\cup C=(A\cup B)\cup C=A\cup(B\cup C),$ $A\cap B\cap C=(A\cap B)\cap C=A\cap(B\cap C)$
Distributivity	$A\cap(B\cup C)=(A\cap B)\cup(A\cap C),$ $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$
Idempotence	$A\cup A=A,\ A\cap A=A$
Absorption	$A\cup(A\cap B)=A,\ A\cap(A\cup B)=A$
Absorption of complement	$A \cup \left(\overline{A} \cap B\right) = A \cup B$ $A \cap \left(\overline{A} \cup B\right) = A \cap B$
Abs. by X and ∅	$A\cup X=X,\ A\cap\varnothing=\varnothing$
Identity	$A\cup\varnothing=A,\ A\cap X=A$
Law of contradiction	$A \cap \overline{A} = \varnothing$
Law of excl. middle	$A \cup \overline{A} = X$
DeMorgan's laws	$\overline{A \cap B} = \overline{A} \cup \overline{B}$ $\quad$ $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Fuzzy Sets

It is important for us to understand the transition from crisp sets to fuzzy sets. As stated earlier the fuzzy sets are more general representation of crisp sets as it is a as a part of its fuzzy set.

3.1 From Classical Sets to Fuzzy Sets :

Let X be the universe of discourse and $x \in X$.

Then a classical set A can be defined as

$$\mathrm{A} = \{x \mid x\ meets\ cetain\ conditions\ \ and\ x \in \mathrm{X}\}$$

For example if we write a set of positive integers below 20 and more than 15. Here the universe of discourse X *is set of all positive integers i.e.* I^+ *and the classical set* A *is as follows:*

$$\mathrm{A} = \{x | 15 < x < 20\, , x \in \mathrm{X}\} \tag{3.1}$$

Hence the classical set is represented by $\mathrm{A} = \{16,17,18,19\}$

If we look at the above classical set A, it is clear that any of the elements in A either completely belongs to A or completely does not belong to A.

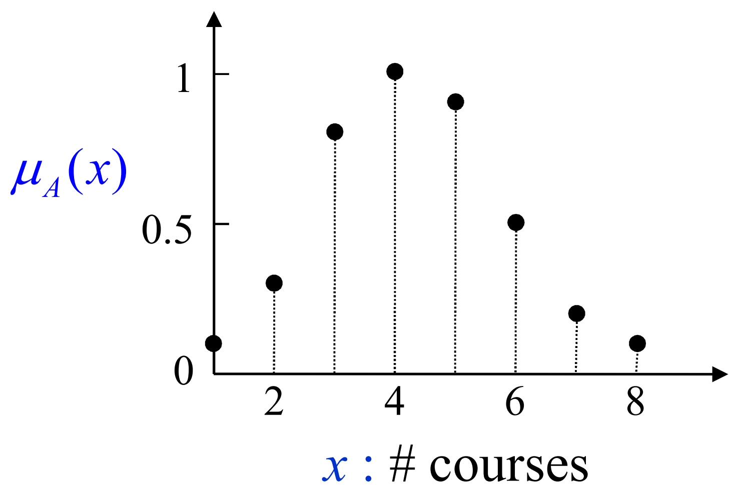
For the same classical set A, in other words we may say that the membership value of any element that belongs to the classical set A is one (True) only. Similarly the elements that are not present in the classical set A, have their corresponding membership values zero (False). This is because classical sets are based on binary logic i.e. 1 (True) or (False) only and there is no possible case in between for any of the elements contained in classical set A.

Since Fuzzy theory is based on multi-valued logic (i.e. not only the true or false) so every element in a fuzzy set A will be assigned its membership value based on its belongingness (i.e. completely belongs, completely does not belong or partially belongs etc.). Hence in Fuzzy sets, membership values are and associated with every element and defined as below.

Example (Discrete Universe)

$$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$$ — # courses a student may take in a semester.

$$A = \left\{ \begin{matrix} (1, 0.1) & (2, 0.3) & (3, 0.8) & (4, 1) \\ (5, 0.9) & (6, 0.5) & (7, 0.2) & (8, 0.1) \end{matrix} \right\}$$ — appropriate courses taken



Alternative Representation:

$$A = 0.1/1 + 0.3/2 + 0.8/3 + 1.0/4 + 0.9/5 + 0.5/6 + 0.2/7 + 0.1/8$$

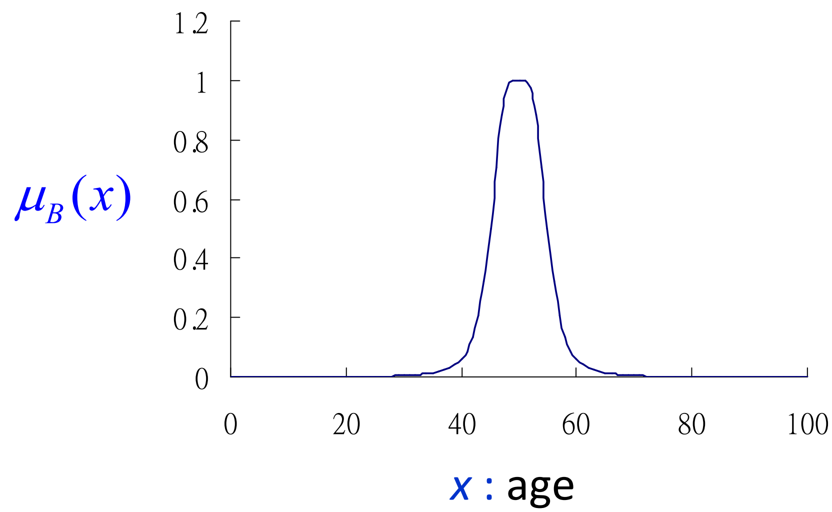
Example (Continuous Universe)

U : the set of positive real numbers —— possible ages

$$B = \left\{ (x, \mu_B(x)) \,\middle|\, x \in U \right\}$$

$$\mu_B(x) = \frac{1}{1 + \left(\dfrac{x-50}{5} \right)^4}$$

about 50 years old



Alternative Representation:

$$B = \int_{R+} \frac{1}{1+\left(\frac{x-50}{5}\right)^4} \Big/ x$$

Alternative Notation

$$A = \left\{ (x, \mu_A(x)) \middle| x \in U \right\}$$

U : discrete universe $\Rightarrow$ $A = \sum_{x_i \in U} \mu_A(x_i) / x_i$

U : continuous universe $\Rightarrow$ $A = \int_U \mu_A(x) / x$

Note that $\sum$ and integral signs stand for the union of membership grades; " $/$ " stands for a marker and does not imply division.

Fuzzy Set-Theory Operations

- Subset

$$A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \quad \forall x \in U$$

- Complement

$$\overline{A} = U - A \Leftrightarrow \mu_{\overline{A}}(x) = 1 - \mu_A(x)$$

- Union

$$C = A \cup B \Leftrightarrow \mu_C(x) = \max(\mu_A(x), \mu_B(x)) = \mu_A(x) \vee \mu_B(x)$$

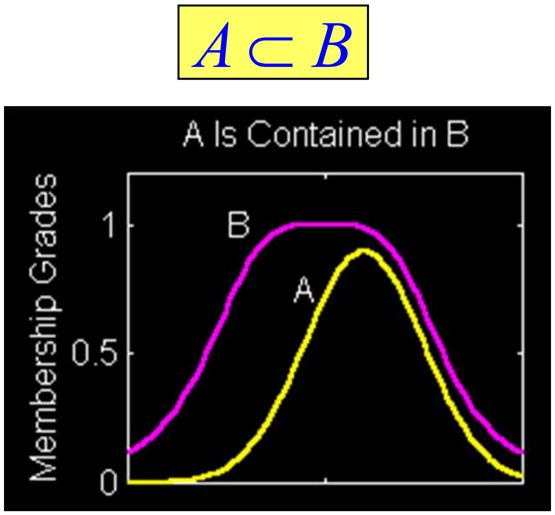
- Intersection

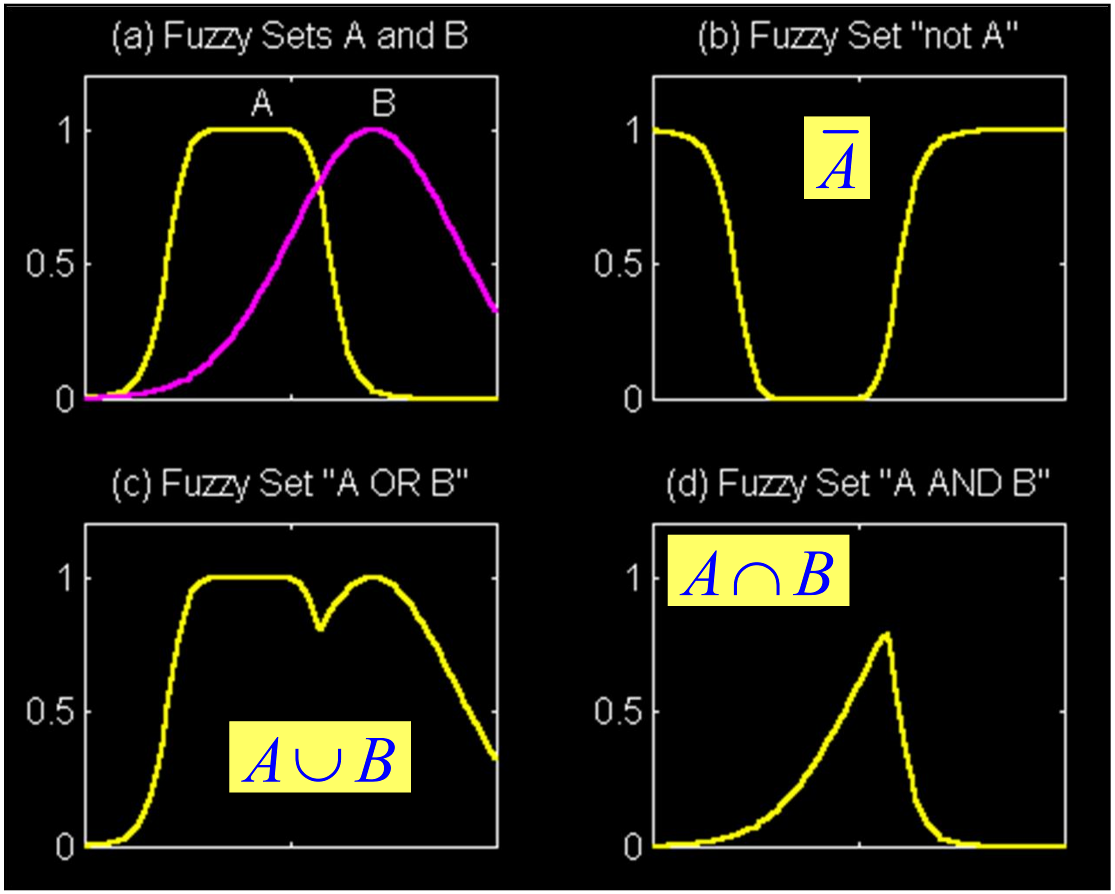
$$C = A \cap B \Leftrightarrow \mu_C(x) = \min(\mu_A(x), \mu_B(x)) = \mu_A(x) \wedge \mu_B(x)$$

Fuzzy Logic Operations

- NOT:
  - If Fuzzy Statement A is *m* true, then the statement "Not A" is *(1.0 – m)* true (where *m* is a number between 0.0 and 1.0 inclusive).
  - Equivalent Set Theory operation: If an object A has *m* membership in Fuzzy Set S, then it must have membership *(1.0 – m)* in Fuzzy Set Not-S.
- AND:
  - If Fuzzy Statement A is *m* true, and Fuzzy Statement B is *n* true, then the Fuzzy Statement "A and B" is *k* true, where *k* = min(*m*,*n*). (Here, *m*, *n* and *k* are numbers between 0.0 and 1.0 inclusive.)
- OR:
  - If Fuzzy Statement A is *m* true, and Fuzzy Statement B is *n* true, then the Fuzzy Statement "A or B" is *k* true, where *k* = max(*m*,*n*). (Here, *m*, *n* and *k* are numbers between 0.0 and 1.0 inclusive.)

Set-Theoretic Operations





Properties

Involution $\overline{\overline{A}} = A$

Commutativity
$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associativity
$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributivity
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Idempotence
$$A \cup A = A$$
$$A \cap A = A$$

Absorption
$$A \cup (A \cap B) = A$$
$$A \cap (A \cup B) = A$$

De Morgan's laws
$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

Properties

- The following properties are *invalid* for fuzzy sets:
  - The laws of contradiction

$$A \cap \overline{A} = \varnothing$$

  - The laws of exclude middle

$$A \cup \overline{A} = U$$

Fuzzy Set Math Operations

- $kA = \{k\mu_A(x), \forall x \in X\}$

  Let k =0.5, and

  A = {0.5/a, 0.3/b, 0.2/c, 1/d}

  then

  kA = {0.25/a, 0.15/b, 0.1/c, 0.5/d}

- $A^m = \{\mu_A(x)^m, \forall x \in X\}$

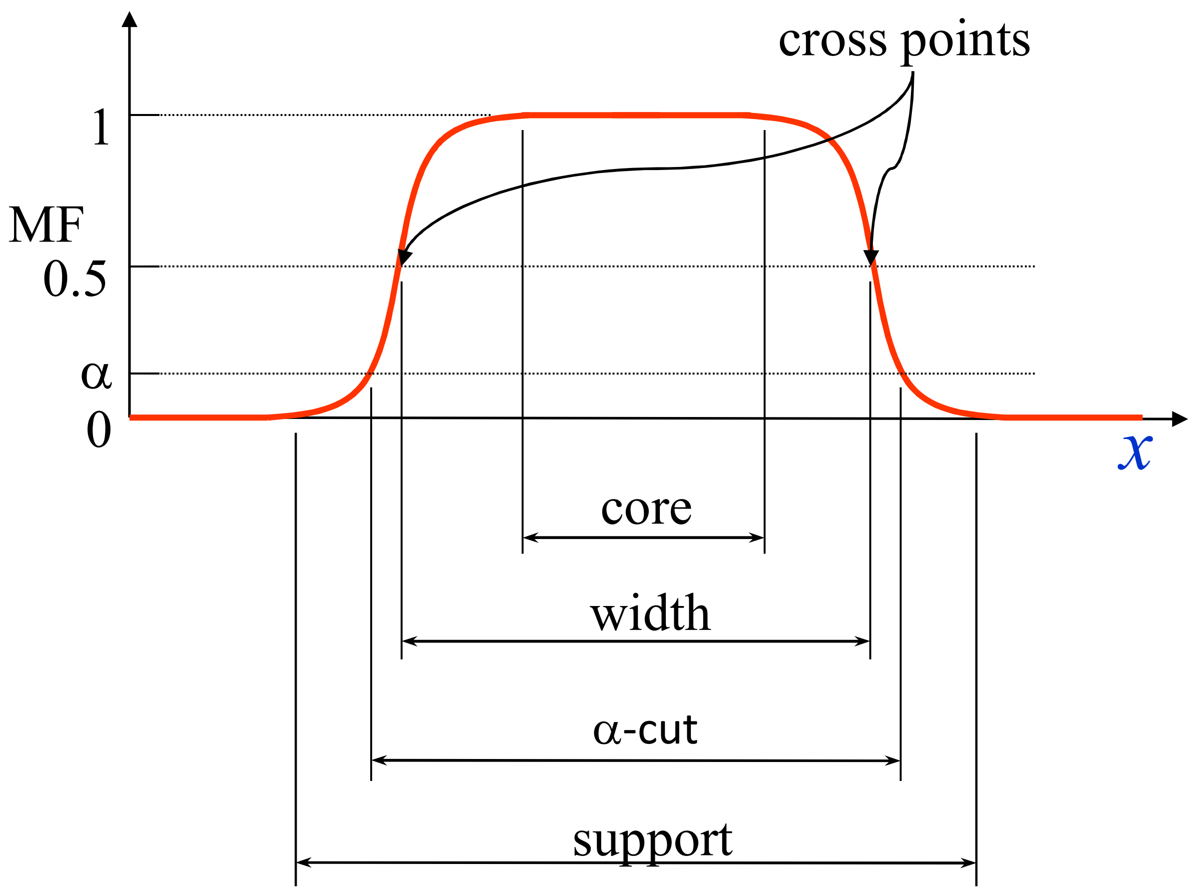
  Let m =2, and

  A = {0.5/a, 0.3/b, 0.2/c, 1/d}

  then

  A^m = {0.25/a, 0.09/b, 0.04/c, 1/d}

MF Terminology



Exercises

For

$A = \{0.2/a, 0.4/b, 1/c, 0.8/d, 0/e\}$

$B = \{0/a, 0.9/b, 0.3/c, 0.2/d, 0.1/e\}$

calculate the following:

- Support, Core and cardinality of A and B
- Complement for *A*, complement of B
- Union and Intersection of A and B,
- the new set $C = A^2$
- the new set $D = 0.5 \times B$

Solutions

A = {0.2/a, 0.4/b, 1/c, 0.8/d, 0/e}
B = {0/a, 0.9/b, 0.3/c, 0.2/d, 0.1/e}

<u>Support</u>
Supp(A) = {a, b, c, d}
Supp(B) = {b, c, d, e}

<u>Core</u>
Core(A) = {c}
Core(B) = {}

<u>Cardinality</u>
Card(A) = 0.2 + 0.4 + 1 + 0.8 + 0 = 2.4
Card(B) = 0 + 0.9 + 0.3 + 0.2 + 0.1 = 1.5

<u>Complement</u>
Comp(A) = {0.8/a, 0.6/b, 0/c, 0.2/d, 1/e}
Comp(B) = {1/a, 0.1/b, 0.7/c, 0.8/d, 0.9/e}

Solutions

A = {0.2/a, 0.4/b, 1/c, 0.8/d, 0/e}
B = {0/a, 0.9/b, 0.3/c, 0.2/d, 0.1/e}

<u>Union</u>

$A \cup B$ = {0.2/a, 0.9/b, 1/c, 0.8/d, 0.1/e}

<u>Intersection</u>

$A \cap B$ = {0/a, 0.4/b, 0.3/c, 0.2/d, 0/e}

<u>$C=A^2$</u>

C = {0.04/a, 0.16/b, 1/c, 0.64/d, 0/e}

<u>$D = 0.5 \times B$</u>

D = {0/a, 0.45/b, 0.15/c, 0.1/d, 0.05/e}

<u>$E = A_{0.5}$</u>

E = {c, d}

Crisp and Fuzzy Relations

Crisp relations

To understand the fuzzy relations, it is better to discuss first crisp relation.

Suppose, A and B are two (crisp) sets. Then Cartesian product denoted as $A \times B$ is a collection of order pairs, such that

$A \times B = \{(a, b) | a \in A \text{ and } b \in B\}$

Note :

(1) $A \times B \neq B \times A$

(2) $|A \times B| = |A| \times |B|$

(3)$A \times B$ provides a mapping from $a \in A$ to $b \in B$.

The mapping so mentioned is called a relation.

Example 1:

Consider the two crisp sets A and B as given below. $A = \{ 1, 2, 3, 4\}$
$B = \{3, 5, 7\}$.

Then, $A \times B = \{(1, 3), (1, 5), (1, 7), (2, 3), (2, 5), (2, 7), (3, 3), (3, 5), (3, 7), (4, 3), (4, 5), (4, 7)\}$

Let us define a relation R as $R = \{(a, b) | b = a + 1, (a, b) \in A \times B\}$

Then, $R = \{(2, 3), (4, 5)\}$ in this case.

We can represent the relation R in a matrix form as follows.

$$R = \begin{array}{c c} & \begin{array}{ccc} 3 & 5 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

Operations on crisp relations

Suppose, $R(x, y)$ and $S(x, y)$ are the two relations define over two crisp sets $x \in A$ and $y \in B$

Union:

$$R(x, y) \cup S(x, y) = max(R(x, y), S(x, y));$$

Intersection:

$$R(x, y) \cap S(x, y) = min(R(x, y), S(x, y));$$

Complement:

$$\overline{R(x, y)} = 1 - R(x, y)$$

Example: Operations on crisp relations

Example:
Suppose, $R(x, y)$ and $S(x, y)$ are the two relations define over two crisp sets $x \in A$ and $y \in B$

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};$$

Find the following:

1. $R \cup S$
2. $R \cap S$
3. $\overline{R}$

Composition of two crisp relations

Given R is a relation on X, Y and S is another relation on Y, Z. Then $R \circ S$ is called a composition of relation on X and Z which is defined as follows.

$$R \circ S = \{(x, z) | (x, y) \in R \text{ and } (y, z) \in S \text{ and } \forall y \in Y\}$$

Max-Min Composition

Given the two relation matrices R and S, the max-min composition is defined as $T = R \circ S$;

$$T(x, z) = max\{min\{R(x, y), S(y, z) \text{ and } \forall y \in Y\}\}$$

Example:

Given

$X = \{1,3,5\}; Y = \{1,3,5\}; R = \{(x,y) | y = x+2\}; S = \{(x,y) | x < y\}$

Here, R and S is on $X \times Y$.

Thus, we have

$R = \{(1,3),(3,5)\}$

$S = \{(1,3),(1,5),(3,5)\}$

$$\text{R=} \begin{array}{cc} & \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \text{ and S=} \begin{array}{cc} & \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\text{Using max-min composition } R \circ S\text{=} \begin{array}{cc} & \begin{array}{ccc} 1 & 3 & 5 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

Fuzzy relations

- Fuzzy relation is a fuzzy set defined on the Cartesian product of crisp set $X_1, X_2, ..., X_n$
- Here, n-tuples $(x_1, x_2, ..., x_n)$ may have varying degree of memberships within the relationship.
- The membership values indicate the strength of the relation between the tuples.

Example:

$X = \{$ typhoid, viral, cold $\}$ and $Y = \{$ running nose, high temp, shivering $\}$

The fuzzy relation R is defined as

$$\begin{array}{c} \\ typhoid \\ viral \\ cold \end{array} \begin{array}{c} \begin{array}{ccc} runningnose & hightemperature & shivering \end{array} \\ \left[\begin{array}{ccc} 0.1 & 0.9 & 0.8 \\ 0.2 & 0.9 & 0.7 \\ 0.9 & 0.4 & 0.6 \end{array}\right] \end{array}$$

Fuzzy Cartesian product

Suppose
A is a fuzzy set on the universe of discourse X with $\mu_A(x)|x \in X$

B is a fuzzy set on the universe of discourse Y with $\mu_B(y)|y \in Y$

Then $R = A \times B \subset X \times Y$; where R has its membership function given by $\mu_R(x, y) = \mu_{A \times B}(x, y) = min\{\mu_A(x), \mu_B(y)\}$

Example :

$A = \{(a_1, 0.2), (a_2, 0.7), (a_3, 0.4)\}$and $B = \{(b_1, 0.5), (b_2, 0.6)\}$

$$R = A \times B = \begin{array}{c c} & \begin{array}{cc} b_1 & b_2 \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ a_3 \end{array} & \begin{bmatrix} 0.2 & 0.2 \\ 0.5 & 0.6 \\ 0.4 & 0.4 \end{bmatrix} \end{array}$$

Operations on Fuzzy relations

Let R and S be two fuzzy relations on $A \times B$.

Union:

$$\mu_{R \cup S}(a, b) = max\{\mu_R(a, b), \mu_S(a, b)\}$$

Intersection:

$$\mu_{R \cap S}(a, b) = min\{\mu_R(a, b), \mu_S(a, b)\}$$

Complement:

$$\mu_{\overline{R}}(a, b) = 1 - \mu_R(a, b)$$

Composition

$$T = R \circ S$$
$$\mu_{R \circ S} = max_{y \in Y}\{min(\mu_R(x, y), \mu_S(y, z))\}$$

Operations on Fuzzy relations: Examples

Example:

$X = (x_1, x_2, x_3)$; Y = (y_1, y_2); Z = (z_1, z_2, z_3);

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.9 \\ 0.8 & 0.6 \end{bmatrix} \end{array}$$

$$S = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.6 & 0.4 & 0.7 \\ 0.5 & 0.8 & 0.9 \end{bmatrix} \end{array}$$

$$R \circ S = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.5 \\ 0.5 & 0.8 & 0.9 \\ 0.6 & 0.6 & 0.7 \end{bmatrix} \end{array}$$

$\mu_{R \circ S}(x_1, y_1) = max\{min(x_1, y_1), min(y_1, z_1), min(x_1, y_2), min(y_2, z_1)\}$
$= max\{min(0.5, 0.6), min(0.1, 0.5)\} = max\{0.5, 0.1\} = 0.5$ and so on.

Fuzzy relation : An example

Consider the following two sets P and D, which represent a set of paddy plants and a set of plant diseases. More precisely

$P = \{P_1, P_2, P_3, P_4\}$ a set of four varieties of paddy plants
$D = \{D_1, D_2, D_3, D_4\}$ of the four various diseases affecting the plants

In addition to these, also consider another set $S = \{S_1, S_2, S_3, S_4\}$ be the common symptoms of the diseases.

Let, R be a relation on $P \times D$, representing which plant is susceptible to which diseases, then R can be stated as

$$R = \begin{array}{c c} & \begin{array}{cccc} D_1 & D_2 & D_3 & D_4 \end{array} \\ \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} & \begin{bmatrix} 0.6 & 0.6 & 0.9 & 0.8 \\ 0.1 & 0.2 & 0.9 & 0.8 \\ 0.9 & 0.3 & 0.4 & 0.8 \\ 0.9 & 0.8 & 0.4 & 0.2 \end{bmatrix} \end{array}$$

Also, consider T be the another relation on $D \times S$, which is given by

$$S = \begin{array}{c} \\ D_1 \\ D_2 \\ D_3 \\ D_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.7 & 0.9 \\ 1.0 & 1.0 & 0.4 & 0.6 \\ 0.0 & 0.0 & 0.5 & 0.9 \\ 0.9 & 1.0 & 0.8 & 0.2 \end{bmatrix} \end{array}$$

Obtain the association of plants with the different symptoms of the disease using **max-min composition.**

Hint: Find $R \circ T$, and verify that

$$R \circ S = \begin{array}{c} \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{array} \begin{array}{c} \begin{array}{cccc} S_1 & S_2 & S_3 & S_4 \end{array} \\ \begin{bmatrix} 0.8 & 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.8 & 0.9 \\ 0.8 & 0.8 & 0.7 & 0.9 \end{bmatrix} \end{array}$$

Fuzzy relation : Another example

Let, $R = x$ is relevant to y

and $S = y$ is relevant to z

be two fuzzy relations defined on $X \times Y$ and $Y \times Z$, respectively, where $X = \{1, 2, 3\}$,$Y = \{\alpha, \beta, \gamma, \delta\}$ and $Z = \{a, b\}$.
Assume that R and S can be expressed with the following relation matrices :

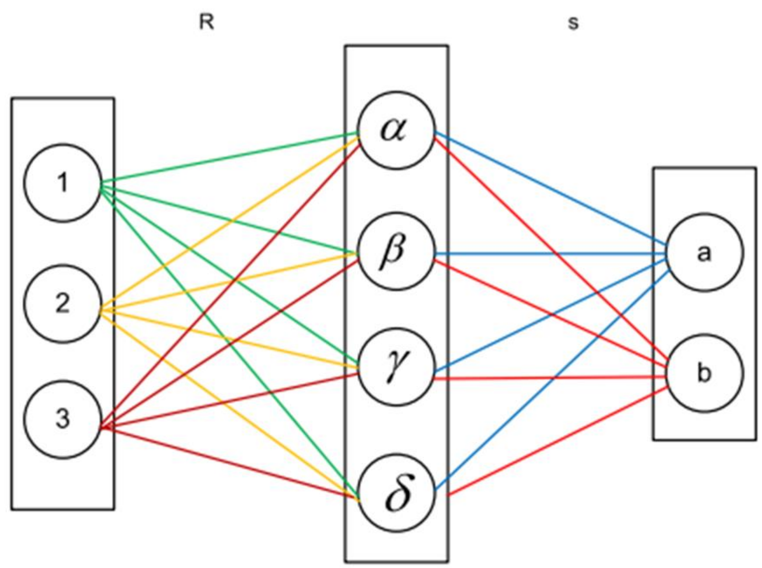
$$R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} \alpha & \beta & \gamma & \delta \end{array} \\ \begin{bmatrix} 0.1 & 0.3 & 0.5 & 0.7 \\ 0.4 & 0.2 & 0.8 & 0.9 \\ 0.6 & 0.8 & 0.3 & 0.2 \end{bmatrix} \end{array} \text{ and}$$

$$S = \begin{array}{c} \\ \alpha \\ \beta \\ \gamma \\ \delta \end{array} \begin{array}{c} \begin{array}{cc} a & b \end{array} \\ \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.3 \\ 0.5 & 0.6 \\ 0.7 & 0.2 \end{bmatrix} \end{array}$$

Now, we want to find $R \circ S$, which can be interpreted as a derived fuzzy relation x **is relevant to** z**.**

Suppose, we are only interested in the degree of relevance between $2 \in X$ and $a \in Z$. Then, using max-min composition,

$$\mu_{R \circ S}(2, a) = max\{(0.4 \wedge 0.9), (0.2 \wedge 0.2), (0.8 \wedge 0.5), (0.9 \wedge 0.7)\}$$
$$= max\{0.4, 0.2, 0.5, 0.7\} = 0.7$$



Example:
Consider the fuzzy sets $\tilde{A}$ and $\tilde{B}$ defined on the interval $X = [0,5]$ of real numbers, by the membership grade functions.

$$\mu_{\tilde{A}}(x) = \frac{x}{x+1}, \quad \mu_{\tilde{B}}(x) = 2^{-x}$$

Determine the Mathematical formulae and graphs of the membership grade functions of each following sets:

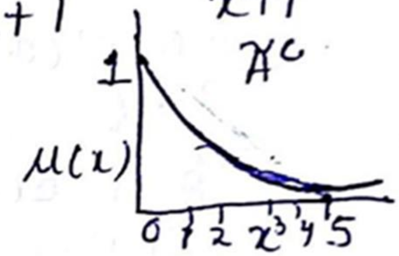
(a) A^c, B^c

(b) $A \cup B$

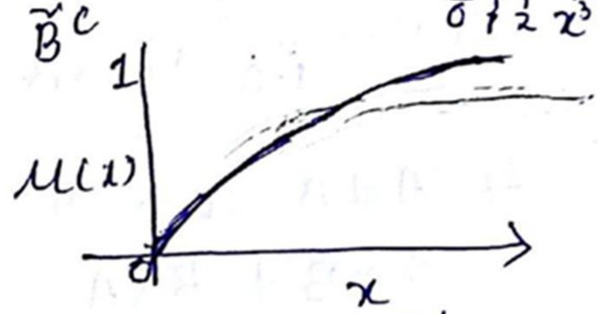
(c) $A \cap B$

(d) $(A \cup B)^c$

Solution: $\mu_{\tilde{A}^c}(x) = 1 - \mu_{\tilde{A}}(x) = 1 - \dfrac{x}{x+1} = \dfrac{1}{x+1}$

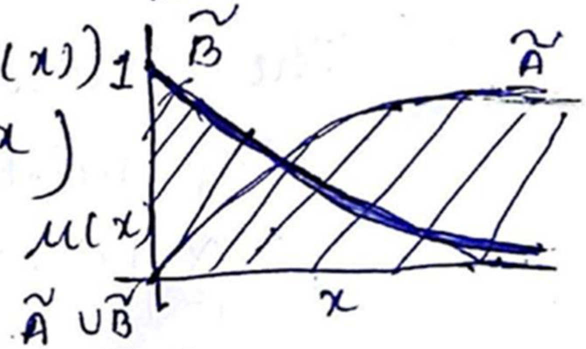


$$\mu_{\tilde{B}^c}(x) = 1 - \mu_{\tilde{B}}(x)$$
$$= 1 - 2^{-x}$$
$$= \frac{2^x - 1}{2^x}$$



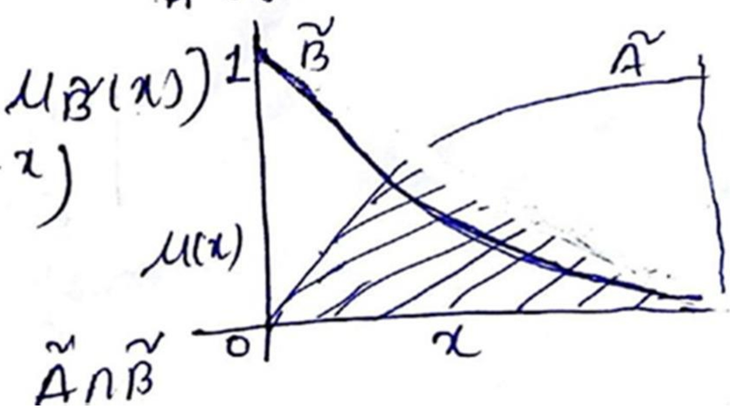
(b) $\mu_{\tilde{A} \cup \tilde{B}}(x) = \max(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$

$$= \max\left(\frac{x}{x+1}, 2^{-x}\right)$$



(c) $\mu_{\tilde{A} \cap \tilde{B}}(x) = \min(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x))$

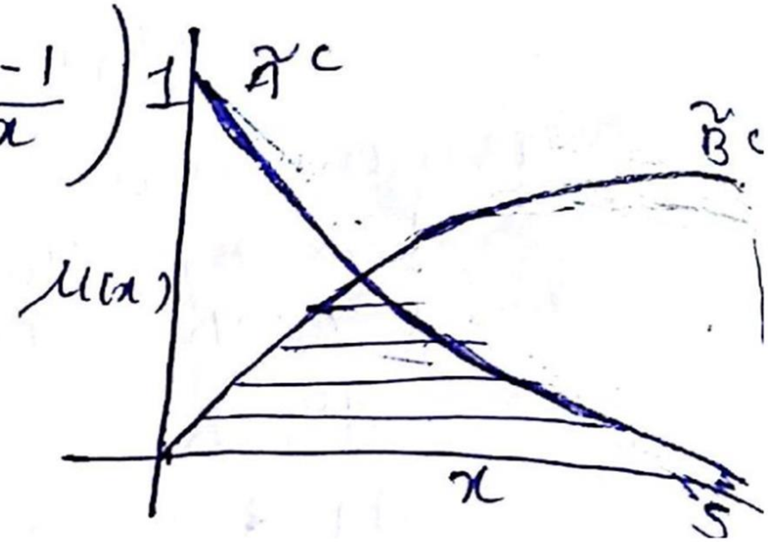
$$= \min\left(\frac{x}{x+1}, 2^{-x}\right)$$



(d) $\mu_{(\tilde{A} \cup \tilde{B})^c}(x) = \mu_{\tilde{A}^c \cap \tilde{B}^c}(x)$ (Demorgan's)

$$= \min\left(\mu_{\tilde{A}^c}(x), \mu_{\tilde{B}^c}(x)\right)$$

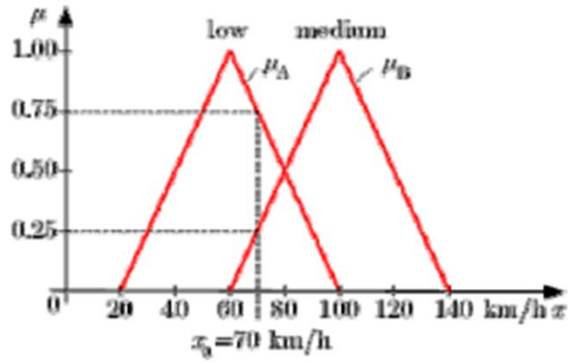
$$= \min\left(\frac{1}{x+1}, \frac{2^x - 1}{2^x}\right)$$



$$(\tilde{A} \cup \tilde{B})^c = \tilde{A}^c \cap \tilde{B}^c$$

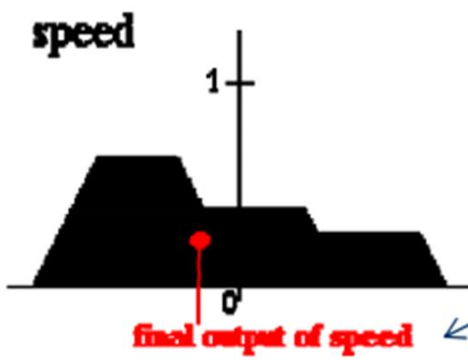
FUZZY-TO-CRISP CONVERSIONS

Fuzzification: Making a crisp quantity fuzzy.



Assignment of membership functions is the process of fuzzification

Defuzzification: Making a fuzzy quantity crisp.



Calculation of a crisp value from a fuzzy value

FROM FUZZY SETS TO CRISP SETS

LAMBDA-CUTS for fuzzy sets

$\underset{\sim}{A}$: a fuzzy set A

A_λ: Lambda-Cut set of A

A_λ: $\{x \mid \mu_{\underline{A}}(x) \geq \lambda\}$ where $0 \leq \lambda \leq 1$

The set A_λ is a crisp set.

Example: X={a,b,c,d,e,f} and $\underset{\sim}{A}=\{\frac{1}{a}+\frac{0.9}{b}+\frac{0.6}{c}+\frac{0.3}{d}+\frac{0.01}{e}+\frac{0}{f}\}$

$\lambda = 1 \quad \rightarrow A_1=\{a\}$;

$\lambda = 0.8 \rightarrow A_{0.8}=\{a,b\}$;

$\lambda = 0.6 \rightarrow A_{0.6}=\{a,b,c\}$;

$\lambda = 0^+ \rightarrow A_{0+}=\{a,b,c,d,e\}$;

$\lambda = 0 \quad \rightarrow A_0=\{a,b,c,d,e,f\} = X$

Fuzzy notation of λ-Cut Sets:

$\underset{\sim}{A}_{0.6}=\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{0}{d}+\frac{0}{e}+\frac{0}{f}\}$

$\underset{\sim}{A}_{0.25}=\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}+\frac{0}{e}+\frac{0}{f}\}$

LAMBDA-CUT SET PROPERTIES

1. $(\underset{\sim}{A} \cup \underset{\sim}{B})_\lambda = A_\lambda \cup B_\lambda$
2. $(\underset{\sim}{A} \cap \underset{\sim}{B})_\lambda = A_\lambda \cap B_\lambda$
3. $(\bar{\underset{\sim}{A}})_\lambda \neq \bar{A}_\lambda$ except for λ=0.5
4. For any $\lambda \leq \alpha$ where $0 \leq \alpha \leq 1$, it is true that $A_\alpha \subseteq A_\lambda$ where A_0=X

LAMBDA-CUTS FOR FUZZY RELATIONS

$\underset{\sim}{R}$: A fuzzy relation

R_λ: λ-cut relation of $\underset{\sim}{R}$.

$R_\lambda=\{(x,y) \mid \mu_{\underset{\sim}{R}}(x,y)\geq\lambda\}$ for $0\leq\lambda\leq1$

FUZZY-TO-CRISP RELATIONS

Example: $R = \begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0.4 \\ 0 & 0.4 & 1 \end{bmatrix}$

$$\lambda = 1 \quad \rightarrow \mathbf{R}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \qquad \lambda = 0.25 \rightarrow \mathbf{R}_{0.25} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix};$$

$$\lambda = 0.5 \rightarrow \mathbf{R}_{0.5} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \qquad \lambda = 0 \quad \rightarrow \mathbf{R}_0 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

FUZZY-TO-CRISP RELATIONS

PROPERTIES:

1. $(\underset{\sim}{R} \cup \underset{\sim}{S})_\lambda = R_\lambda \cup S_\lambda$
2. $(\underset{\sim}{R} \cap \underset{\sim}{S})_\lambda = R_\lambda \cap S_\lambda$
3. $(\underset{\sim}{\bar{R}})_\lambda \neq \bar{R}_\lambda$
4. For any $\lambda \leq \alpha$ where $0 \leq \alpha \leq 1$, then $R_\alpha \subseteq R_\lambda$.